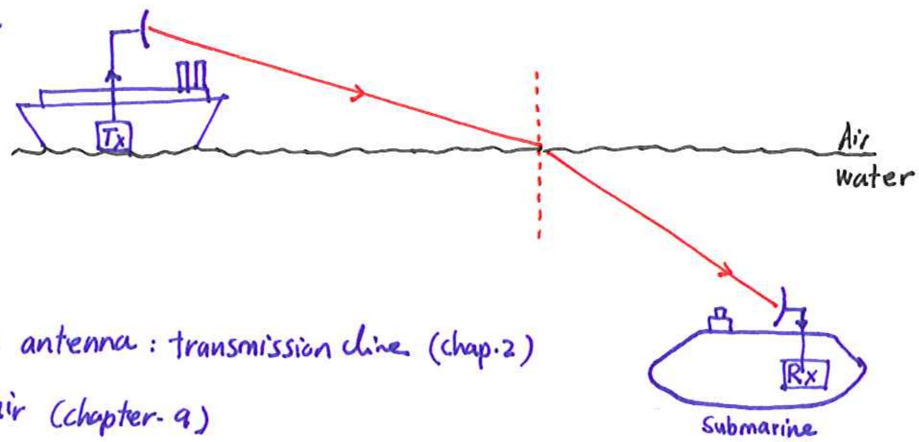
Chapter 8

EM Waves at boundaries

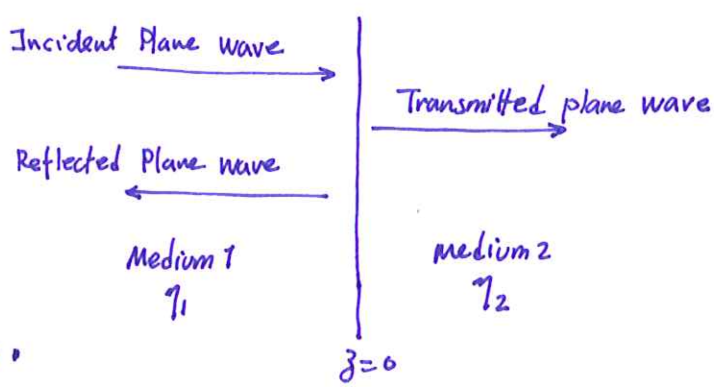
The communication system between a ship and a submarine is a good example for a complete communication via EM wave.



1) From transmitter (Tx) to the antenna: transmission line (chap.2)
2) Radiation from antenna to air (chapter.9)
3) Travel in air: wave propagation in lossless media (chap.7)
4) Reflection and transmission at air/water boundary (chap.8)
5) Travel in water: wave propagation in lossy media (chap.7)
6) Reception at submarine's antenna (chap.9)
7) From antenna to the receiver: transmission line (chap.2)

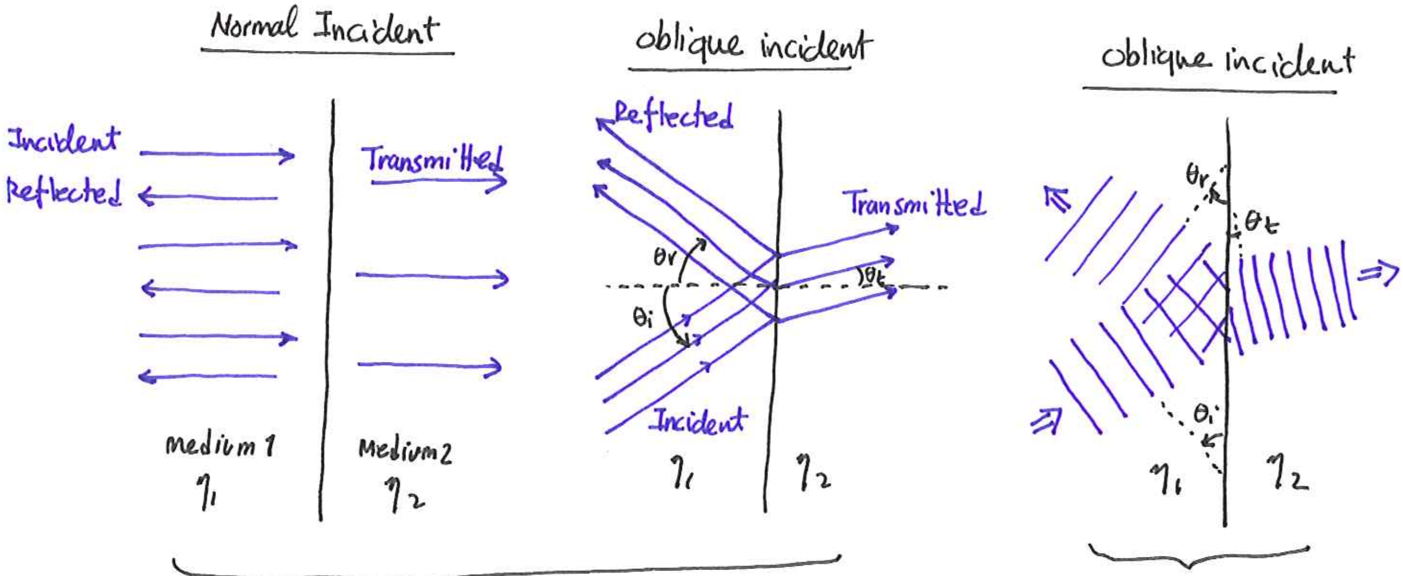
Wave Reflection and Transmission at Normal Incident

A uniform plane wave propagating in an unbounded medium, when encounters a boundary, is reflected from and transmitted through the boundary.



The boundary conditions are same as the ones we developed for voltage and current in a transmission line in chapter 2.

Boundary between Lossless Media



For Normal Incident:

Incident Wave:

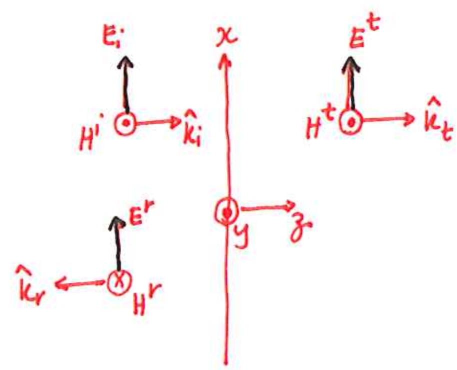
$$\begin{cases} \tilde{E}^i(z) = \hat{x}\, E_0^i\, e^{-jk_1 z} \\ \tilde{H}^i(z) = \hat{z} \times \dfrac{\tilde{E}^i(z)}{\eta_1} = \hat{y}\, \dfrac{E_0^i}{\eta_1}\, e^{-jk_1 z} \end{cases}$$

We also know:

$$\begin{cases} k_1 = \omega\sqrt{\mu_1 \varepsilon_1} & \eta_1 = \sqrt{\dfrac{\mu_1}{\varepsilon_1}} \\ k_2 = \omega\sqrt{\mu_2 \varepsilon_2} & \eta_2 = \sqrt{\dfrac{\mu_2}{\varepsilon_2}} \end{cases}$$

Reflected wave:

$$\begin{cases} \tilde{E}^r(z) = \hat{x}\, E_0^r\, e^{jk_1 z} \\ \tilde{H}^r(z) = (-\hat{z}) \times \dfrac{\tilde{E}^r(z)}{\eta_1} = -\hat{y}\, \dfrac{E_0^r}{\eta_1}\, e^{jk_1 z} \end{cases}$$



Transmitted wave:

$$\begin{cases} \tilde{E}^t(z) = \hat{x}\, E_0^t\, e^{-jk_2 z} \\ \tilde{H}^t(z) = \hat{z} \times \dfrac{\tilde{E}^t(z)}{\eta_2} = \hat{y}\, \dfrac{E_0^t}{\eta_2}\, e^{-jk_2 z} \end{cases}$$

Total E and H in each medium:

Midium 1

$$\begin{cases} \vec{\tilde{E}}_1(z) = \vec{\tilde{E}}^i(z) + \vec{\tilde{E}}^r(z) = \hat{x}\left(E_0^i e^{-jk_1 z} + E_0^r e^{jk_1 z}\right) \\ \vec{\tilde{H}}_1(z) = \vec{\tilde{H}}^i(z) + \vec{\tilde{H}}^r(z) = \hat{y}\, \dfrac{1}{\eta_1}\left(E_0^i e^{-jk_1 z} - E_0^r e^{jk_1 z}\right) \end{cases}$$

Medium II $\begin{cases} \tilde{E}_2(z) = \tilde{E}^t(z) = \hat{x}\, E_0^t\, e^{-jk_2 z} \\ \tilde{H}_2(z) = \tilde{H}^t(z) = \hat{y}\, \dfrac{E_0^t}{\eta_2}\, e^{-jk_2 z} \end{cases}$

Boundary Condition

1) Tangential components of E are equal at interface ($z=0$)

2) Since there is no interface current, tangential components of H are also equal at $z=0$

$$\Rightarrow \begin{cases} \vec{\tilde{E}}_1(0) = \vec{\tilde{E}}_2(0) \rightarrow E_0^i + E_0^r = E_0^t \\ \vec{\tilde{H}}_1(0) = \vec{\tilde{H}}_2(0) \rightarrow \dfrac{E_0^i}{\eta_1} - \dfrac{E_0^r}{\eta_1} = \dfrac{E_0^t}{\eta_2} \end{cases} \Rightarrow \begin{cases} E_0^r = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1} E_0^i = \Gamma E_0^i \\ E_0^t = \dfrac{2\eta_2}{\eta_1 + \eta_2} E_0^i = \tau E_0^i \end{cases}$$

Reflection Coeffic.: $\Gamma = \dfrac{E_0^r}{E_0^i} = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$ (Normal incident)

Transmission Coeffic.: $\tau = \dfrac{E_0^t}{E_0^i} = \dfrac{2\eta_2}{\eta_2 + \eta_1}$ (Normal incident)

$\rightarrow$ $\boxed{\tau = 1 + \Gamma}$ (Normal incident)

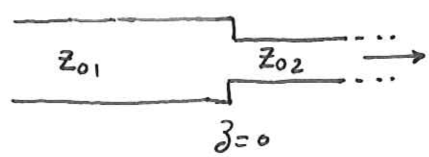
For nonmagnetic media:

$$\eta_1 = \frac{\eta_0}{\sqrt{\varepsilon_{r_1}}} \;\&\; \eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_{r_2}}} \rightarrow \boxed{\Gamma = \frac{\sqrt{\varepsilon_{r_1}} - \sqrt{\varepsilon_{r_2}}}{\sqrt{\varepsilon_{r_1}} + \sqrt{\varepsilon_{r_2}}}}$$

Normal incident
Nonmagnetic media.

Transmission-Line Analogue

The voltage reflection coefficient for two transmission lines connected at $z=0$ is:



$\Gamma = \dfrac{Z_{02} - Z_{01}}{Z_{02} - Z_{01}}$ here Z_0 is replaced with η for unbounded transmission.

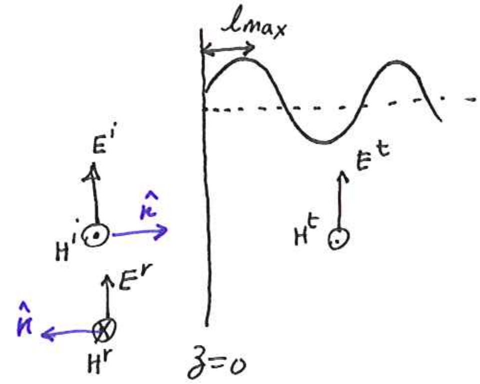
By analogy with the transmission-line case, we define a standing-wave ratio in medium 1 as:

$$\boxed{S = \frac{|\tilde{E}_1|_{max}}{|\tilde{E}_1|_{min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}}$$

If $\eta_2 = \eta_1 \Rightarrow |\Gamma| = 0 \Rightarrow S = 1$ (minimum value for S)

If $\eta_2 = 0$ (medium 2 be a perfect conductor) $\Rightarrow \Gamma = -1 \Rightarrow |\Gamma| = 1 \Rightarrow S = \infty$ (maximum S)
(Similar to a short circuited trans. line)

The distance l_{max} where E becomes maximum from the boundary is given by same equations derived for transmission line:



$$\boxed{\begin{aligned} -z = l_{max} &= \frac{\theta_r + 2n\pi}{2k_1} = \frac{\theta_r \lambda_1}{4\pi} + \frac{n\lambda_1}{2} \\ &\begin{cases} n = 1, 2, \dots & \theta_r < 0 \\ n = 0, 1, 2, \dots & \theta_r \geq 0 \end{cases} \end{aligned}}$$

where $\lambda_1 = \frac{2\pi}{k_1}$ and $\Gamma = |\Gamma| e^{j\theta_r}$ $(-\pi \leq \theta_r \leq \pi)$

The spacing between a maximum and the nearest minimum is $\frac{\lambda_1}{4}$.

The E_{min}'s (minima) occur at: $l_{min} = \begin{cases} l_{max} + \lambda_1/4 & \text{if } l_{max} < \frac{\lambda_1}{4} \\ l_{max} - \lambda_1/4 & \text{if } l_{max} \geq \frac{\lambda_1}{4} \end{cases}$

Power Flow in Lossless Media

Average Power flow in medium 1 is: $S_{av_1} = \hat{z} \frac{|E_0^i|^2}{2\eta_1} (1 - |\Gamma|^2)$

Proof: $S_{av_1}(z) = \frac{1}{2} \mathrm{Re}\left[\vec{\tilde{E}}_1(z) \times \tilde{H}_1^*(z) \right]$

$$= \frac{1}{2} \mathrm{Re}\left[\hat{x} E_0^i (e^{-jk_1 z} + \Gamma e^{jk_1 z}) \times \hat{y} \frac{E_0^{i*}}{\eta_1} (e^{jk_1 z} - \Gamma^* e^{-jk_1 z}) \right]$$

$$= \hat{z} \frac{|E_0^i|^2}{2\eta_1} (\underbrace{1}_{\text{incident part}} - \underbrace{|\Gamma|^2}_{\text{reflected part}})$$

$S_{av_1} = S_{av}^i + S_{av}^r \rightarrow$ $S_{av}^i = \hat{z} \frac{|E_0^i|^2}{2\eta_1}$

$S_{av}^r = -\hat{z} |\Gamma|^2 \frac{|E_0^i|^2}{2\eta_1}$

The average transmitted power is: $S_{av_2}(z) = \frac{1}{2} \mathrm{Re}\left[\tilde{E}_2(z) \times \tilde{H}_2^*(z) \right]$

$$= \frac{1}{2} \mathrm{Re}\left[\hat{x} \tau E_0^i e^{-jk_2 z} \times \hat{y} \tau^* \frac{E_0^{i*}}{\eta_2} e^{jk_2 z} \right]$$

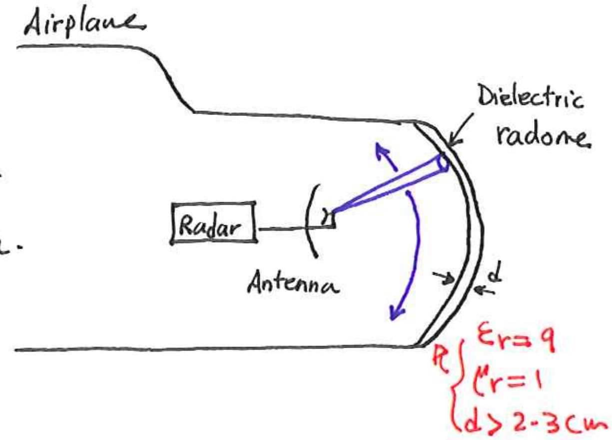
$$= \hat{z} |\tau|^2 \frac{|E_0^i|^2}{2\eta_2}$$

From the expressions for τ and Γ we can also show that:

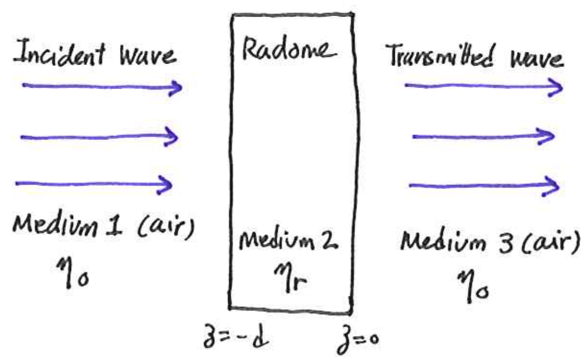
$$\left.\begin{aligned} \Gamma &= \frac{\eta_2-\eta_1}{\eta_2+\eta_1} \\ \tau &= 1+\Gamma \end{aligned}\right\} \quad \frac{\tau^2}{\eta_2} = \frac{1-\Gamma^2}{\eta_1} \quad \text{(lossless media)} \implies S_{av_1} = S_{av_2} \text{ as expected from conservation of energy}$$

Example

A 10-GHz radar is used in an airplane head. The antenna is mounted on a gimbal behind a dielectric radome. The EM beam has small cross section compared with the radome area and the wave can be approximated planar. If the radome material is a lossless dielectric with $\mu_r = 1$ and $\varepsilon_r = 9$, choose its thickness d so that the radome appears transparent to the radar. The radom thickness must be $> 2\text{-}3$ cm for mechanical integrity.



Transparent radome means $\to$ $\Gamma = 0$ at $z = -d$ (no reflection) so the whole power is transmitted through the radome.

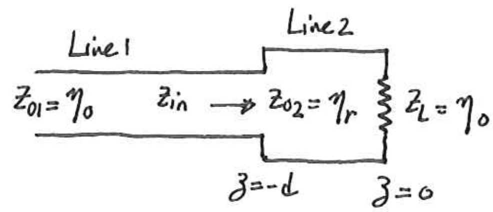


$Z_L = \eta_0 \implies \Gamma(z=-d) = 0$ if $Z_{in} = \eta_0$

$\implies d = n\dfrac{\lambda_2}{2} \qquad n = 0, 1, 2, \dots$

$f = 10\,\text{GHz} \to \lambda_0 = \dfrac{c}{f} = \dfrac{3\times10^8}{10\times10^9} = 3\text{ cm} \to \lambda_2 = \dfrac{\lambda_0}{\sqrt{\varepsilon_r}} = \dfrac{3\text{ cm}}{3} = 1\text{ cm}$

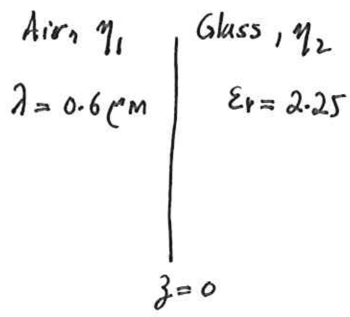
|||



$\to d = n\dfrac{1\text{ cm}}{2}$ choosing $n = 5 \implies$

$d = 2.5\text{ cm}$.

Example Yellow light Incident Upon a Glass Surface



A beam of yellow light is incident into glass. The light wavelength is $\lambda = 0.6\,\mu m$. The glass $\varepsilon_r = 2.25$. Determine:

(a) the locations of maxima of E in air

(b) the standing wave ratio

(c) the fraction of the incident power transmitted into the glass

Solution: (a)

$$\eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \simeq 120\pi\ (\Omega)$$

$$\eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} \cdot \frac{1}{\sqrt{\varepsilon_r}} \simeq \frac{120\pi}{\sqrt{2.25}} = 80\pi\ (\Omega)$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{80\pi - 120\pi}{80\pi + 120\pi} = -0.2$$

$$\Gamma = -0.2 = 0.2e^{j\pi} \rightarrow \theta_r = \pi \rightarrow l_{max} = \frac{\theta_r \lambda_1}{4\pi} + n\frac{\lambda_1}{2} = \frac{\lambda_1}{4} + n\frac{\lambda_1}{2} \quad (n = 0, 1, 2, \cdots)$$

$$= \frac{0.6}{4} + n\frac{0.6}{2} = 0.15 + 0.3n\ \mu m$$

(b)

$$S = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.2}{1-0.2} = 1.5$$

(c)

$$\frac{S_{av2}}{S_{av}^i} = \frac{\tau^2 \frac{|E_0^i|^2}{2\eta_2}}{\frac{|E_0^i|^2}{2\eta_1}} = \tau^2 \frac{\eta_1}{\eta_2} = 1 - |\Gamma|^2 = 1 - 0.2^2 = 0.96 \text{ or } 96\%$$

$$\left(\frac{\tau^2}{\eta_2} = \frac{1-\Gamma^2}{\eta_1}\right)$$

Boundary between Lossy Media

In this case the intrinsic impedance is complex: $\eta_c = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu}{\varepsilon' - j\varepsilon''}}$

The propagation constant is also: $\gamma = \alpha + j\beta$ where α indicates the loss. (See eqns. 7.66 in the book)

In Medium 1:

$$\begin{cases} \vec{\tilde{E}}_1(z) = \hat{x}\, E_0^i \left(e^{-\gamma_1 z} + \underline{\underline{\Gamma}} e^{\gamma_1 z}\right) \\ \vec{\tilde{H}}_1(z) = \hat{y}\, \frac{E_0^i}{\eta_{c1}} \left(e^{-\gamma_1 z} - \underline{\underline{\Gamma}} e^{\gamma_1 z}\right) \end{cases}$$

In Medium 2:

$$\begin{cases} \vec{\tilde{E}}_2(z) = \hat{x}\, \underline{\underline{\tau}}\, E_0^i\, e^{-\gamma_2 z} \\ \vec{\tilde{H}}_2(z) = \hat{y}\, \underline{\underline{\tau}}\, \frac{E_0^i}{\eta_{c2}}\, e^{-\gamma_2 z} \end{cases}$$

where:

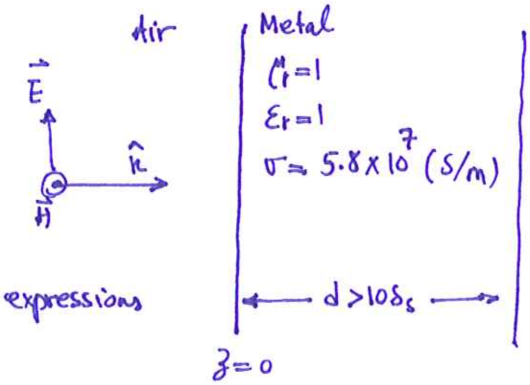
$$\boxed{\Gamma = \frac{\eta_{c2} - \eta_{c1}}{\eta_{c2} + \eta_{c1}} \qquad \tau = 1 + \Gamma = \frac{2\eta_{c2}}{\eta_{c2} + \eta_{c1}}}$$

And of course: $\gamma_1 = \alpha_1 + j\beta_1$

$\gamma_2 = \alpha_2 + j\beta_2$

Example Normal Incidence on a Metal Surface

A 1-GHz x-polarized TEM wave is incident on a metal surface as shown. The metal thickness is more than several skin depth.
If the amplitud of $\vec{E}$ is $12 \left(\frac{mV}{m}\right)$, obtain $\vec{E}$ and $\vec{H}$ expressions in air.



Solution: $\Gamma = \dfrac{\eta_{c2} - \eta_0}{\eta_{c2} + \eta_0}$ So we need to find η_{c2}.

For a metal we can use the approximate relation (Eq. 7.77c in the book):

$$\eta_{c_2} = \sqrt{\frac{\mu}{\varepsilon_c}} \simeq (1+j)\sqrt{\frac{\pi f \mu}{\sigma}}$$

Note: This is because for a metal $\frac{\varepsilon''}{\varepsilon'} \gg 1$. To double check:

$$\frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega \varepsilon_r \varepsilon_0} = \frac{5.8 \times 10^7}{2\pi \times 10^9 \times 1 \times \varepsilon_0} = 10^9 \gg 1$$ So we can use the above approximation.

otherwise we would use the general form of $\eta_{c_2} = \sqrt{\frac{\mu}{\varepsilon_c}} = \sqrt{\frac{\mu}{\varepsilon' - j\varepsilon''}} = \sqrt{\frac{\mu_r \mu_0}{\varepsilon_r \varepsilon_0 - j\frac{\sigma}{\omega}}}$

$$\eta_{c_2} = (1+j)\left(\frac{\pi \times 10^9 \times 4\pi \times 10^{-7}}{5.8 \times 10^7}\right)^{1/2} = 8.25(1+j) \quad (m\Omega)$$

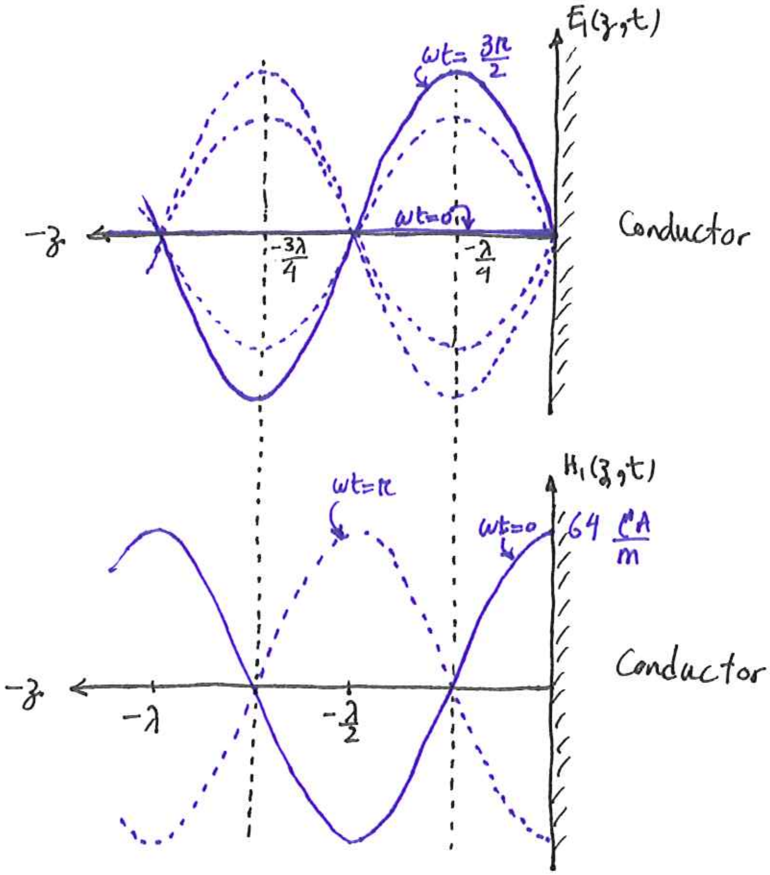
η_{c_2} compared to $\eta_0 = 120\pi = 377\ \Omega$ is very small $\eta_{c2} \ll \eta_0 \Rightarrow$

$$\Gamma = \frac{\eta_{c_2} - \eta_0}{\eta_{c_2} + \eta_0} \simeq \frac{-\eta_0}{\eta_0} = -1 \Rightarrow$$

$$k_1 = \frac{\omega}{c} = \frac{2\pi \times 10^9}{3 \times 10^8} = \frac{20\pi}{3}$$

$$\begin{cases} \tilde{E}_1(z) = \hat{x}\, E_0^i \left(e^{-jk_1 z} - e^{jk_1 z}\right) = -\hat{x}\, j 2 E_0^i \sin k_1 z = -\hat{x}\, j\, 24 \sin k_1 z \\ \tilde{H}_1(z) = \hat{y}\, \frac{E_0^i}{\eta_1}\left(e^{-jk_1 z} + e^{jk_1 z}\right) = \hat{y}\, 2\frac{E_0^i}{\eta_1}\cos k_1 z = \hat{y}\, \frac{24}{377}\cos k_1 z = \hat{y}\, 0.064 \cos k_1 z \end{cases}$$

$$\rightarrow \begin{cases} \vec{E}_1(z,t) = \hat{x}\, 24 \sin\left(\frac{20\pi z}{3}\right) \sin(2\pi \times 10^9 t) \quad \left(\frac{mV}{m}\right) \\ \vec{H}_1(z,t) = \hat{y}\, 64 \cos\left(\frac{20\pi z}{3}\right) \cos(2\pi \times 10^9 t) \quad \left(\frac{\mu A}{m}\right) \end{cases}$$


$E_1(z,t)$
$\omega t = \frac{3\pi}{2}$
$\omega t = 0$
$-z$
$-\frac{3\lambda}{4}$
$-\frac{\lambda}{4}$
Conductor
$H_1(z,t)$
$\omega t = \pi$
$\omega t = 0$
$64\ \frac{mA}{m}$
$-z$
$-\lambda$
$-\frac{\lambda}{2}$
Conductor